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Cyclotron resonance of magnetopolarons in anisotropic parabolic quantum dots

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The interaction of electrons confined in a quasi-zero-dimensional anisotropic parabolic quantum dot and longitudinal-optical (LO) phonons, placed in a perpendicular magnetic field is calculated. Analytical and numerical results are presented for the anisotropy and polaron corrections to the Landau levels of an isotropic parabolic quantum dot.

Through advances in high-resolution submicrometer lithography the fabrication of semiconductor nanostructures in which confinement of the electronic motion in narrow quantum well wires (QWW) and quantum dots (QD) is realizable. With the quasi-one-dimensional (Q1D) and quasi-zero-dimensional (Q0D) systems, electron gases of all four dimensionalities from 3D to Q0D are artificially realized by technological means.

The energy levels of an electron in a strong magnetic field are quantized into Landau levels. If the electron is in a polar semiconductor it also interacts with the optical phonons. Hence, the Landau levels are modified by polaronic effects.

The polaron mass is experimentally determined by cyclotron resonance. In such an experiment the separation of adjacent Landau levels is measured as a function of the magnetic field B. Hence, in polar semiconductors the cyclotron resonance frequency $\omega_C^* = eB/m_C^*$, with m_C^* the polaron cyclotron mass, is affected by the interaction of the electrons with the optical phonons. For the 3D [1–3] and Q2D [3–5] polaron, considerably much work has been done. Two different situations are commonly distinguished in 3D and Q2D systems: the nonresonant magnetopolaron in low magnetic fields and the resonant magnetopolaron in quantizing magnetic fields when the cyclotron energy approximately equals the optical phonon energy. But in Q1D and Q0D systems it depends on the confinement potential if a resonant case is possible or not [6,7].

In this paper we investigate magnetopolarons in QDs with an anisotropic parabolic confinement potential in the lateral direction. Because the electron is confined within the QD the magnetopolaron is a bound magnetopolaron. The electron–phonon correction will be calculated within second-order perturbation theory for arbitrary magnetic fields. The unperturbed system, a single electron confined in a zero thickness (x, y)-plane along the z-direction at $z = 0$ and confined in a lateral anisotropic parabolic quantum well potential in the (x, y)-plane in the presence of a quantizing perpendicular magnetic field $\boldsymbol{B} = (0, 0, B)$ is described by the Hamiltonian

$$H_e = \frac{1}{2m_e}(\boldsymbol{p} + e\boldsymbol{A})^2 + V(\boldsymbol{x}) + \frac{g^*}{2}\mu_B B\sigma_z \tag{1}$$

with m_e the effective conduction band-edge mass, $\boldsymbol{A}$ the vector potential with $\boldsymbol{B} = \nabla \times \boldsymbol{A}$ and $V(\boldsymbol{x}) = V(x, y) + V(z)$, the confining potential. The lateral anisotropic parabolic quantum well

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potential in the (x, y)-plane is given by

$$V(x, y) = m_e(\Omega_x^2 x^2 + \Omega_y^2 y^2)/2 .$$

$\mu_B = e\hbar/(2m_0)$ denotes Bohr's magneton, with m_0 the free electron mass, g^* is the effective spin-splitting (Landé) factor and σ_z denotes the Pauli matrix. In the following we ignore the Zeeman term ($g^* \to 0$). For the vector potential A we use the symmetric gauge $A = (-y, x, 0)B/2$. The one-electron Hamiltonian $H_e = H_e^0 + H_e^1$ can be separated in an isotropic part

$$H_e^0 = (p + eA)^2/2m_e + m_e\Omega^2 r^2/2$$

with $\Omega = \Omega_y$ and $r^2 = x^2 + y^2$, and the deviation from it $H_e^1 = m_e\Omega_\Delta^2 x^2/2$ with $\Omega_\Delta^2 = \Omega_x^2 - \Omega_y^2$. The eigenenergies

$$\mathscr{E}^0_{N_r,m} = \hbar\tilde{\omega}_C(2N_r + |m| + 1) + \hbar\omega_C m/2$$

with $\tilde{\omega}_C = ((\omega_C/2)^2 + \Omega^2)^{1/2}$, $\omega_C = eB/m_e$, $N_r = 0, 1, 2, \ldots, m = 0, \pm1, \pm2, \ldots$ and the single-particle wave functions $\Psi^{(0)}_{N_r m}(r, \varphi)$ of the isotropic part H_e^0 are well-known [7]. According to the strict confinement assumed in the z-direction $|\varphi(z)|^2 = \delta(z)$ is valid. To obtain the single-particle energies and eigenstates of H_e we assume that H_e^1 is a perturbation of H_e^0. Hence, the wave function of H_e is given by

$$\Psi_i(r, \varphi) = \sum_{N_r,m} A^i_{N_r m} \Psi^{(0)}_{N_r m}(r, \varphi)$$

and the eigenenergies are determined from the algebraic system of equations

$$\sum_{N_r,m} A^i_{N_r m}\{(\mathscr{E}_i - \mathscr{E}^{(0)}_{N_r m})\delta_{N_r N_r'}\delta_{mm'} - \langle N_r' m'|H_e^1|N_r m\rangle\} = 0 . \qquad (2)$$

According to the spatial symmetry of the problem this system of algebraic equations splits into two separate ones: one for even m and the other for odd m. It is suitable to denote the new Landau levels $\mathscr{E}_i$ with $i = (N_x, N_y)$ which correspond to the quantum numbers of harmonic oscillators along the x- and y-direction for vanishing magnetic field.

In fig. 1 the first Landau levels $\mathscr{E}_{N_x N_y}$ of the anisotropic parabolic QD calculated using eq. (2) are plotted. It is to be seen that at vanishing magnetic field the $(N+1)$-fold degeneracy ($N = 2N_r + |m|$) of the states $\Psi^{(0)}_{N_r m}(r, \varphi)$ which exist for an isotropic parabolic potential is lifted. This is caused by the loss of the radial symmetry in the (x, y)-plane according to the anisotropic parabolic potential. In fig. 1(a) the deviation of the equipotential lines from circles is very weak and hence the energy spectrum is very similar to that of the isotropic parabolic potential. In fig. 1(b) the equipotential lines are more eccentric ellipses and hence the energy spectrum deviates appreciably from that of the isotropic case. From both figures it is to be seen that with increasing

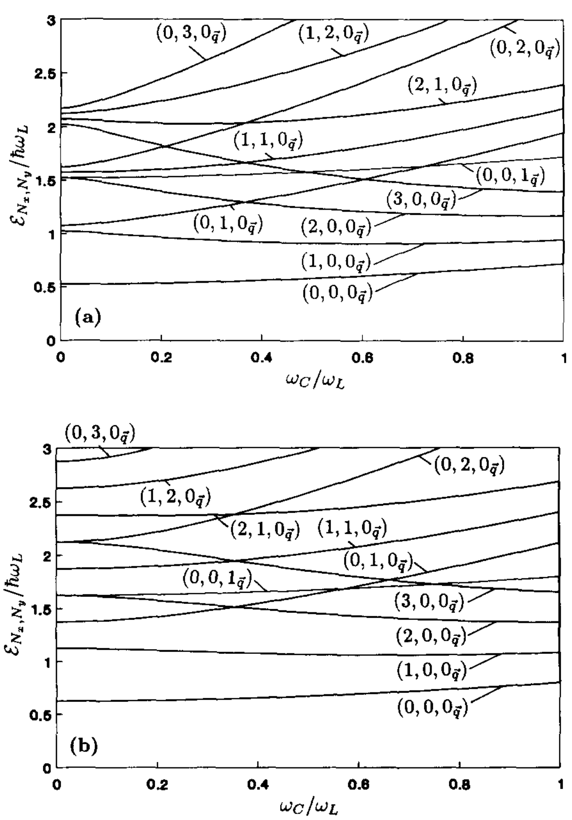

Fig. 1. The first Landau levels (N_x, N_y, n_q) as a function of the magnetic field of an anisotropic parabolic GaAs–$Ga_{0.75}Al_{0.25}As$ QD (thick solid line) with $\Omega/\omega_L = 0.5$ (nq: number of LO phonons). The thin solid line corresponds to the unperturbed level $(0, 0, 1_q)$. In (a) the Landau levels are plotted for $\Omega_x = 1.1\Omega_y$ and in (b) for $\Omega_x = 1.5\Omega_y$.

magnetic field a similar crossing behaviour of the Landau levels occurs as in the case of the isotropic parabolic potential.

Our interest is directed to QDs generated by nanostructured gate electrodes via the field effect. Hence, the optical phonons interacting with the electrons are these of the original layered semiconductor structure. Neglecting the effects of interface phonons [8,9] the Hamiltonian of the electron–phonon interaction including only 3D bulk longitudinal-optical (LO) phonons [10] reads

$$H_{\mathrm{ep}} = \left(\frac{4\pi\alpha r_{\mathrm{p}}(\hbar\omega_{\mathrm{L}})^2}{V_{\mathrm{G}}}\right)^{1/2} \times \sum_{q} \mathrm{e}^{\mathrm{i}qx} \frac{1}{|q|} (a_{\mathrm{L}}(q) + a_{\mathrm{L}}^{+}(-q)) \qquad (3)$$

with α the dimensionless 3D polaron coupling constant, r_{p} the corresponding 3D polaron radius and ω_{L} the frequency of the LO phonons of the semiconductor containing the Q0D confined electrons, respectively. $q = (q_x, q_y, q_z)$ is the 3D wave vector of the 3D bulk LO phonon. The magnetopolaron Hamiltonian is given by

$$H_{\mathrm{p}} = H_{\mathrm{e}} + \sum_{q} \hbar\omega_{\mathrm{L}}(a_{\mathrm{L}}^{+}(q)a_{\mathrm{L}}(q) + \tfrac{1}{2}) + H_{\mathrm{ep}} = H_0 + H_1 . \qquad (4)$$

The first two terms represent the unperturbed electron and LO phonon system, H_0, and $H_1 = H_{\mathrm{ep}}$ is the electron–phonon interaction Hamiltonian. The energy levels of an electron are shifted over $\Delta E_{N_x,N_y}$ by the interaction with the LO phonons:

$$E_{N_x,N_y} = \mathscr{E}_{N_x,N_y} + \Delta E_{N_x,N_y} .$$

Within second-order perturbation theory the energy shift of the level with the quantum numbers N_x, N_y is given by

$$\Delta E_{N_x,N_y} = - \sum_{N_x',N_y'=0}^{\infty} \sum_{q} \frac{|M_{N_y'N_y}^{N_x'N_x}(q)|^2}{\hbar\omega_{\mathrm{L}} + (\mathscr{E}_{N_x',N_y'} - \mathscr{E}_{N_x,N_y}) - \Delta_{N_x,N_y}} . \qquad (5)$$

The matrix element reads

$$|M_{N_y'N_y}^{N_x'N_x}(q)|^2 = \sum_{N_{r1}N_{r2}} \sum_{N_{r3}N_{r4}} \sum_{m_1m_2} \sum_{m_3m_4} A_{N_{r1}m_1}^{N_x'N_y'} (A_{N_{r2}m_2}^{N_x'N_y'})^* A_{N_{r3}m_3}^{N_xN_y} (A_{N_{r4}m_4}^{N_xN_y})^* \times (M_{m_1m_4}^{N_{r1}N_{r4}}(q))^* M_{m_2m_3}^{N_{r2}N_{r3}}(q) \qquad (6)$$

with

$$M_{m'm}^{N_r'N_r}(q) = {}^{(0)}\langle N_r', m', 1_q | H_{\mathrm{ep}} | N_r, m, 0_q \rangle^{(0)} .$$

The ket $|N_r, m, n_q\rangle^{(0)}$ describes an unperturbed state of H_0 for the isotropic parabolic potential, composed of an electron in the level N_r, m and n LO phonons with the momentum $\hbar q$ and the energy $\hbar\omega_{\mathrm{L}}$ which we denote (N_r, m, n_q). The matrix elements $M_{m'm}^{N_r'N_r}(q)$ are calculated in ref. [7]. Here we only consider weakly polar semiconductors with $\alpha \ll 1$, i.e. we are in the weak-coupling limit and so it is sufficient to consider perturbed states containing not more than one LO phonon. The value $\Delta_{N_xN_y}$ depends on the type of the perturbation theory which is used [4]: (i) $\Delta_{N_xN_y} = 0$ leads to the Rayleigh–Schrödinger perturbation theory (RSPT); (ii) $\Delta_{N_xN_y} = \Delta E_{N_xN_y}$ results in the Wigner–Brillouin perturbation theory (WBPT); (iii) $\Delta_{N_xN_y} = \Delta E_{N_xN_y} - \Delta E_{00}^{\mathrm{RSPT}}$ gives an improved Wigner–Brillouin perturbation theory (IWBPT) with $\Delta E_{00}^{\mathrm{RSPT}}$ the weak-coupling electron–phonon correction to the electron ground-state energy calculated within RSPT. For the ground-state $\Delta E_{00}^{\mathrm{IWBPT}} = \Delta E_{00}^{\mathrm{RSPT}}$ is valid. It is well known [4] that the RSPT describes the ground-state correction for $\omega_{\mathrm{C}} \to 0$ quite well, but it fails for the excited states, since it is possible that the denominator vanishes for a certain ω_{C}. This becomes possible if the energy level $(N_x, N_y, 0_q)$ of the state $|N_x, N_y, 0_q\rangle$ crosses the energy level $(0, 0, 1_q)$ of the state $|0, 0, 1_q\rangle$. But the occurrence of a level crossing depends strongly on the relation between the LO phonon frequency ω_{L} and the confinement frequency Ω. This behaviour is very similar to the case of a QWW with parabolic confinement [6] and a QD with isotropic parabolic confinement [7] but different to the 3D and Q2D systems where level crossing always occurs at $N\omega_{\mathrm{C}} = \omega_{\mathrm{L}}$ (N: number of the Landau level). If resonance occurs, the electron–phonon interaction leads to a splitting of the degenerated levels and a pinning to the energy $\hbar\omega_L + \mathscr{E}_{00} + \Delta E_{00}^{\mathrm{RSPT}}$. Only the IWBPT gives the correct pinning behaviour in the weak-coupling limit.

For numerical calculations we have used a nanostructured GaAs–$Ga_{1-x}Al_xAs$ heterostructure (GaAs: $\alpha = 0.07$, $r_p = 3.987$ nm, $\hbar\omega_L = 36.17$ meV, $m_e = 0.06624 m_0$) in which the electrons are confined within GaAs.

The calculated Landau levels for one electron in different anisotropic QDs including polaron effects using IWBPT are plotted in fig. 2. The thin solid lines show the unperturbed levels of an anisotropic parabolic QD $(N_x, N_y, 0_q)$, the thin dashed line the unperturbed level $(0, 0, 1_q)$ and the heavy solid lines are the corresponding perturbed levels which are calculated from eq. (5). From fig. 2 it is apparent that the perturbed levels, the magnetopolaron levels, are (i) shifted to lower energies $\sim\Delta E_{N_xN_y}(\omega_C = 0)$ independent of the magnetic field and (ii) with increasing magnetic field the state $|1, 0, 0_q\rangle$ mixes strongly with $|0, 0, 1_q\rangle$ becoming resonant near the unperturbed level crossing. The Landau levels are repelled from the level $(0, 0, 1_q)$ and pinned to the energy $\hbar\omega_L + \mathscr{E}_{00} + \Delta E_{00}^{\mathrm{RSPT}}$. The crossing point for the unperturbed levels is shifted with increasing eccentricity to lower magnetic fields. Therefore it is possible to measure the resonance phenomena and following the pinning of Landau levels already for low magnetic fields. Figure 2 shows that the polaron correction to the Landau levels at $B = 0$ are different for different quantum numbers (N_x, N_y). This behaviour is different from the well-known 3D and 2D magnetopolaron [3] for which the polaron corrections at $B = 0$ are independent of the quantum numbers, but similar to the polaron corrections to the Landau levels at $B = 0$ for QWWs [6] and QDs [7]. Moreover, one can see that the splitting of the levels $(1, 0, 0_q)$ and $(0, 1, 0_q)$ for small magnetic fields caused by the anisotropic parabolic potential in the (x, y)-plane becomes smaller if the coupling to the LO phonons is considered.

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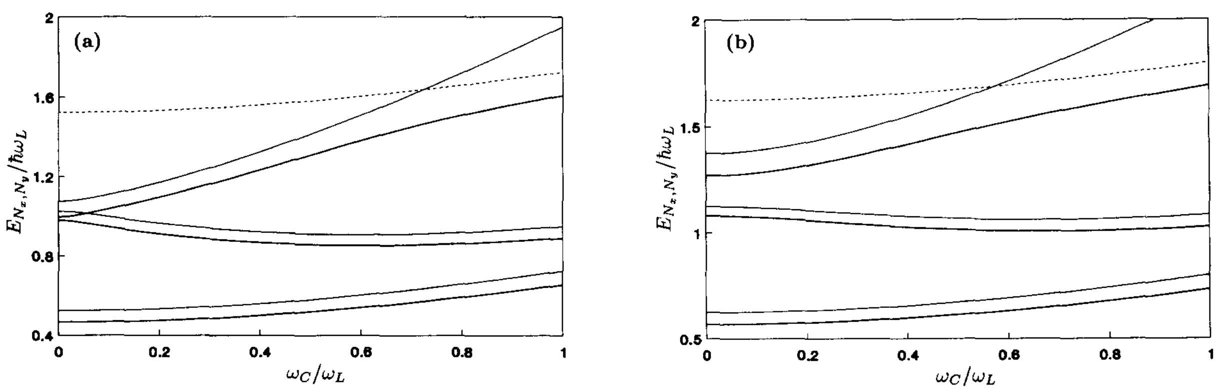

Fig. 2. The first magnetopolaron levels E_{00}, E_{10} and E_{01} (thick solid lines) as a function of the magnetic field in an anisotropic parabolic GaAs–$Ga_{0.75}Al_{0.25}As$ QD with $\Omega/\omega_L = 0.5$ for $\Omega_x = 1.1\Omega_y$ (a) and $\Omega_x = 1.5\Omega_y$ (b). The corresponding unperturbed Landau levels are plotted by thin solid ($(0, 0, 0_q)$, $(0, 1, 0_q)$, $(1, 0, 0_q)$) and dashed ($(0, 0, 1_q)$) lines.